N65-89046
~~X64 11771~~

6p

Code 2A
(NASA TMX-51390)

~~ENCYCLOPEDIA OF EARTH PHYSICS~~

EARTH'S FIGURE

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The term "Figure of the Earth" is sometimes defined as the equipotential shape corresponding to the Earth's external gravitational field, and other times as a simplification thereof convenient for use as a reference model. This article conforms to the latter definition; see "The Geoid" by W. A. Heiskanen in this encyclopedia for a discussion of the irregular variations in the earth's external form.

The simplest geometric and gravitational model to which all variations of the actual earth can be referred as small linear corrections requires four parameters: its mass, M; its equatorial radius, a; its rate of rotation, ω; and either its moment of inertia, C, or its flattening,

$$f = (a-b)/a \tag{1}$$

where b is the polar radius.

The physically most logical model is to specify the moment of inertia, C, and to require that the form of the model be that of the rotating fluid for the given parameters; then to obtain the flattening, f, (Jeffreys, 1959, p.151):

$$1 - \frac{3C}{2Ma^2} = \frac{2}{5}\left[\frac{5\omega^2 a^3}{2kMf}(1-f) - 1\right]^{1/2} + O(f^2) \tag{2}$$

where k is the gravitational constant.

The form of the rotating fluid is not methematically simple, however, so in geodesy it is customary to use as a reference figure the ellipsoid of revolution, which differs by less than 10^{-6} in radius at all latitudes. The reference gravitational field is then calculated under the assumption

Submitted for Publication

that this ellipsoid is an equipotential. Since the mass M is not accurately known, nor was the product kM readily deduced from observations until recent years, it is also customary in geodesy to adopt as fundamental the acceleration of gravity at the equator, γ_e , related to the mass, M, by:

$$kM = a^2 \gamma_e \left[1 - f + \frac{3}{2} m - \frac{15}{14} m f + O(f^3)\right] \qquad (3)$$

where

$$m = \frac{\omega^2 a}{\gamma_e} \qquad (4)$$

The resulting formula for the acceleration of gravity γ at geodetic latitude ϕ is:

$$\gamma = \gamma_e \left[1 + \beta_2 \sin^2 \phi + \beta_4 \sin^2 2\phi + O(f^3)\right] \qquad (5)$$

where

$$\beta_2 = \frac{5}{2} m - f - \frac{17}{14} m f \qquad (6)$$

$$\beta_4 = \frac{1}{8} f^2 - \frac{5}{8} m f \qquad (7)$$

The resulting formula for the gravitational potential V at geocentric latitude ψ and radius r is:

$$V = \frac{kM}{r} \left[1 - J_2 \left(\frac{a}{r}\right)^2 P_2(\sin\psi) - J_4 \left(\frac{a}{r}\right)^4 P_4(\sin\psi) - O(f^3)\right] \qquad (8)$$

where $P_2(\sin\psi)$ and $P_4(\sin\psi)$ are the 2nd and 4th degree Legendre polynomials of sin ψ and:

$$J_2 = \frac{(C-A)}{Ma^2} = \frac{2}{3}f\left[1-\frac{f}{2}\right] - \frac{m}{3}\left[1-\frac{3}{2}m-\frac{2}{7}f\right] + O(f^3) \quad (9)$$

$$J_4 = -\frac{4}{35}f(7f-5m) + O(f^3), \quad (10)$$

where C is the moment of inertia about the polar axis and A is the moment about an equatorial axis.

Numerical values adopted as standard by international scientific organizations and values considered to be current best estimates are given in the table. The most accurately known parameter is the rotation rate, ω, determined by observation of the stars with photographic zenith tubes. Comparison with clocks indicates seasonal variations in the rate of about 1 in 10^8. Comparison with the rate of motion of the moon, as determined by eclipses, indicates a secular decrease of ω of about 2 in 10^{10} per year (Munk and MacDonald, 1960, p.203).

The value of J_2 in (9), and hence of the flattening f, is now most accurately determined from the secular motions of the nodes, (and, to a lesser extent, the perigees) of close satellite orbits. Since these motions are also affected by higher order J_{2n} of the actual earth which are independent quantities of $0(10^{-6})$, a variety of orbital inclinations is required to determine J_2: the lack of such a variety of accurately determined orbits is the principal limitation of the accuracy of J_2. The standard deviations of J_2 and f given in the table are based on the discrepancies between different authors' results. The observed J_4 from these same satellite studies is -1.44×10^{-6}, about 60 percent of the ellipsoidal J_4 in the table.

The determination of the equatorial gravity γ_e comprises two separate parts: (1) the determination of the absolute value of gravity by timing pendulum swings or the fall of an object in the laboratory; and (2) the analysis of gravimetry which gives the values of gravity all over the earth's surface relative to the absolute determination. The inaccuracy of

γ_e is about equally due to discrepancies in different laboratory determinations of absolute gravity and to the scanty and non-uniform distribution of gravimetry.

The equatorial radius a can be determined with comparable accuracy by three methods: (1) by fitting an ellipsoid, or an ellipsoid plus geoid heights computed from gravimetry, to an astro-geodetic geoid; (2) by comparison of the positions of tracking stations determined from satellite orbits with the geoid heights at the same points determined from orbits or gravimetry; and (3) by using a kM determined from satellite orbits in equation (3) with a γ_e determined from terrestrial data. Of these three, (2) and (3) are quite new. The errors in methods (1) and (2) arise primarily from the triangulation systems, and in method (3) from the determination of kM. The value, and its uncertainty, in the table is based on solutions by methods (1) and (2) and their discrepancies.

The product gravitational constant times mass, kM, can be obtained from a, γ_e, f, and m by equation (3). The value in the table was calculated in this manner. Alternatively, it can be determined from the semi-major axis A and mean motion n in Kepler's law for an earth satellite:

$$kM = n^2 A^3 , \tag{11}$$

appropriately modified to take into account secular perturbations and, in the case of the moon, the mass of the satellite. The length A will depend on the velocity of light, in the case of range and range-rate observations, and on the triangulation connecting tracking stations in the case of directional observations. The principal source of error in using the moon is its radius; in using close satellites, it is the distortion of the estimated mean motion by drag and other poorly determined perturbations. Astronomical determinations of kM have in nearly all cases differed from that in the table by less than 1 in 100,000.

The final type of observation of interest is the precession of the earth's axis due to the attraction of the moon for the earth's equatorial bulge, which yields a determination of (C-A)/C of 1/305.51. Use of this quantity with the J_2 determined from satellite orbits in equation (9) obtains a value of C/Ma^2 for use in equation (2). The resulting "hydrostatic" flattening f is 1/300. If the flattening from J_2 is used in (2) to solve for the rotation ω, a rate 3 in 10^3 greater is obtained: equivalent to the rotation 1.5 x 10^7 years ago at the present rate of decrease, and hence an estimate of the time constant for the response of the earth as a whole to stress.

TABLE: PARAMETERS FOR THE FIGURE OF THE EARTH

Parameter	Standard Value	Current Estimate and Standard Deviation
	Observed	
Mean sidereal rotation rate, ω	$.7292115085 \times 10^{-4} sec^{-1}$	$.7292115085 \times 10^{-4} sec^{-1}$
Oblateness coefficient, J_2	.0010921	.0010827 ± .0000002
Equatorial gravity, γ_e	$9.780490 msec^{-2}$	$9.780306 \pm .000013 msec^{-2}$
Equatorial radius, a	6,378,388. meters	6,378,160 ± 15 meters
	Derived	
Centrifugal force/equatorial gravity ratio, eq. (4), m	.0034678	.0034678

Gravitational coefficient, eq. (10), J_4	-.00000234	-.00000234
Flattening, eq. (9), f	1/297.00	1/298.25 ± .03
Gravity coefficient, eq. (6), β_2	.0052884	.0053025 ± .0000003
Gravity coefficient, eq. (7), β_4	-.0000059	-.0000059
Mass x Grav. constant, eq. (3), kM	$3.98632 \times 10^{14} m^3 sec^{-2}$	$3.986026 \pm .000020 \times 10^{14} m^3 sec^{-2}$

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